Chapter 1 Background and Introduction

1.1. Preface

Gradient-index (GRIN) profiles are a part of daily life. Their effects are best known from mirages, as documented almost 200 years ago[1] and explained as a result of temperature variations causing a refractive index variation in the air. Variations of refractive index are also part of living creatures. Exner discovered that insect eyes are composed of rods which act as lenses due to their varying index of refraction.[2] The human eye, as well, has a radial gradient-index profile.[3] And though we see via a natural gradient-index lens, ophthalmic corrective lenses are not currently fabricated using gradient-index materials.

Conventionally, gradient-index lenses have some form of symmetry, often radial symmetry, and have a uniform optical power. They are also typically small in size, less than five millimeters in diameter, have a small index change, $\Delta n<0.03$, and usually spanning a few millimeters. But ophthalmic lenses can have large diameters, tens of millimeters. And to provide good performance, a gradient-index ophthalmic lens may require a large index change, , $\Delta n>0.05$,. Gradient-index glasses are not a likely solution; however gradient-index plastics are. Understanding and control of gradient-index plastic fabrication is necessary, if GRIN ophthalmic lenses are to be made.

Of course, design principles for GRIN ophthalmic lenses are also required. Then building on an understanding of GRIN ophthalmic lens design, the more complex problem of progressive addition lenses (a multifocal lens) can be tackled. Unlike traditional lenses, these are non-symmetric, with varying optical power. New design methods are required for designing such GRIN lenses.

Understanding the fabrication of gradient-index plastics becomes even more valuable, with design methods for non-symmetric, multifocal GRIN lenses. Currently, the index profiles available in GRIN media are limited, and information about GRIN plastics even more so. Gradient-index profile formation must be characterized, if the ultimate goal of arbitrary index profiles is to be achieved.

This dissertation examines these design challenges and presents effective methods for meeting them. This research extends the understanding of GRIN lens design into large-diameter, non-symmetric, multifocal lenses. The design theories presented are also shown to be useful beyond just ophthalmic lenses. And to further the state of the art with GRIN materials, the fabrication of a GRIN copolymer is characterized. In total, this dissertation develops design techniques for gradient-index ophthalmic lenses. It also continues efforts towards large-radius gradient-index optics by analyzing their fabrication in an interesting polymer system.

1.2. Progressive Addition Lens Overview

With the increase of wisdom, comes the decrease of vision: near the age of 45 years old, a person’s eyes lose the ability to accommodate. The eye can no longer change the shape of the lens, to change focus according to viewing distance. This condition is called “Presbyopia”.[4,5] Without the ability to accommodate, the presbyope only has good vision for one viewing distance and requires vision correction for at least some portion of his viewing needs. Because this is a universal problem, the need for effective correction is great.

The most common corrective lens is the bifocal lens, but these cause a power "jump" when the eye gaze traverses the boundary between the two lens regions. Other corrective methods include: soft bifocal contact lenses (which sacrifice image clarity), mono-vision contact lens technique (which sacrifice depth perception), and progressive addition lenses (PALs).

The progressive addition lens is a multifocal lens meant to give the presbyope clear vision at a range of viewing distances. They have a change in power from the upper (distance) region to the lower (near, or reading) region.[6] The lenses are optimized for both optical and physiological parameters. The front surface is typically the progressive surface, and provides both the desired power variation for vision correction and aberration control. An example power gradient for a typical progressive addition lens is shown in Figure 1.1.[7]

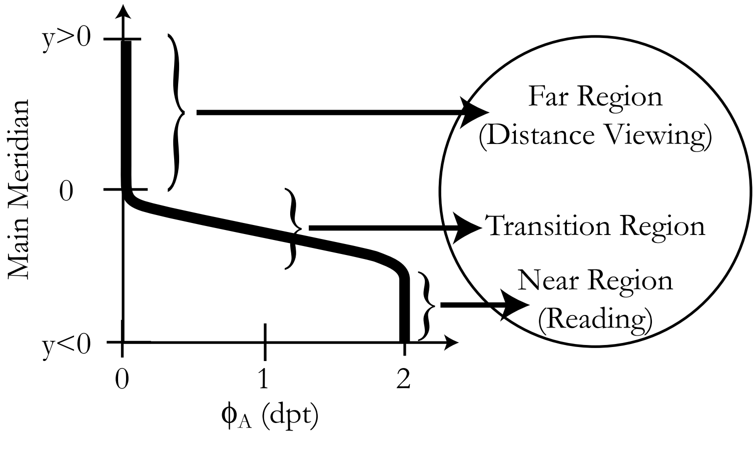

Figure 1.1. Example power progression along vertical axis of PAL

Progressive addition lenses have two primary advantages. First, they appeal to the wearer’s vanity. Unlike bifocals and trifocals, progressive addition lenses do not have a visible boundary between the lens regions. This is generally considered a more attractive design. Perhaps more importantly, they do not betray the wearer’s age. The other advantage is that the smooth transition between power zones alleviates the power jump caused by the abrupt power differences in bifocals and trifocals. The interest in these lenses is shown by the fact that progressive addition lenses currently outsell bifocals.[8-10]

1.3. Historical Background

The first patent on a progressive addition lens was given to Aves[11] in 1907. The power gradient was due to both lens surfaces being shaped so the radius of curvatures in the horizontal direction varied along the lens vertical. In 1920, a French patent was issued for a lens which required only a single surface’s figure to provide the power progression.[12] Another form appears in the 1946 Beach patent.[13] It describes a lens with front surface having both a near power region and a far power region connected by annular zones of power progression.

The first commercially successful progressive lens was designed by Maitenaz in the 1950s.[14] This design evolved through several stages until around 1958, when the now standard form was reached. (This is the form illustrated in Figure 1.1)[14]

The earliest reference to using a gradient-index material in corrective lenses is the 1924 Bugbee patent,[15] which describes a method to make a bifocal lens using gradient-index glass. He describes a design for a homogeneous ophthalmic lens with a small recess. In the recess is fused another glass with a gradient-index. This section provides the higher power, for near vision, in his patent.

The earliest reference to a multifocal plastic lens using GRIN materials is Naujokas' 1969 patent.[16] It describes a process to create GRIN progressive lenses using a polymer diffusion technique. His lens is envisioned to have horizontal regions in the lens with various refractive indices. One region would have a linearly increasing gradient, providing a smooth power transition, created by diffusing one liquid monomer into a different polymer to create a gradient-index polymer. More general uses of gradient-index polymers are proposed by R. S. Moore in his 1973 patent.[17] Ophthalmic lenses are one suggested application, as he references the Naujokas patent.

1.4. Gradient-Index Design

1.4.1. Spherical Gradient

The vast majority of gradient-index lens designs have some degree of symmetry. The earliest gradient-index design is a spherically symmetric lens, the Maxwell fisheye lens. It provides perfect point-to-point imaging within the lens.[18] Luneberg has proposed a spherical lens which can image an external object.[19]

1.4.2. Axial Gradient

More commonly studied and manufactured are the axial and radially-symmetric gradients. In general, the refractive index for these profiles can be written as,

$$n(r,z) = \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} N_{ij} r^{2i} z^{j} , \tag{1.1}$$

where *r* is the radial direction, *z* is the distance along the optical axis, and *n* is the refractive index. The axial gradient index, illustrated in Figure 1.2, is then expressed as,

$$n(z) = N_{00} + N_{01}z + N_{02}z^2 + \cdots \tag{1.2}$$

This index gradient provides additional degrees of freedom in lens design for aberration control. Specifically, it can be used to remove spherical aberration from a spherical lens; with proper lens bending, coma can also be eliminated.[20]

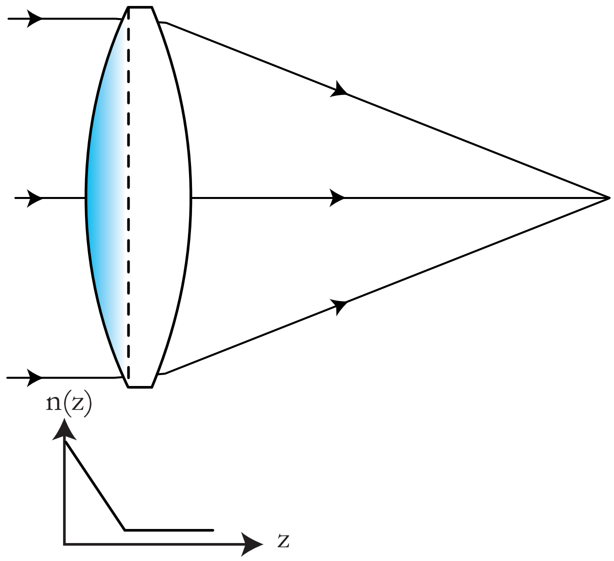

Figure 1.2. Spherical lens with linear, axial gradient. The shaded region represents the index gradient, with darker regions indicating higher refractive indices.

1.4.3. Radial Gradient

The axial gradient provides aberration control; the radially-symmetric gradient can provide optical power. The index profile is generally written as,

$$n(r) = N_{00} + N_{10} r^2 + N_{20} r^4 + \cdots . \qquad \textbf{(1.3)}$$

If there is a quadratic component to the index profile, it creates optical power and can be written as

$$\phi = -2 N_{10} t , \qquad \textbf{(1.4)}$$

where *t* is the lens thickness. The classic example of this effect is the Wood lens[21], illustrated in Figure 1.3.

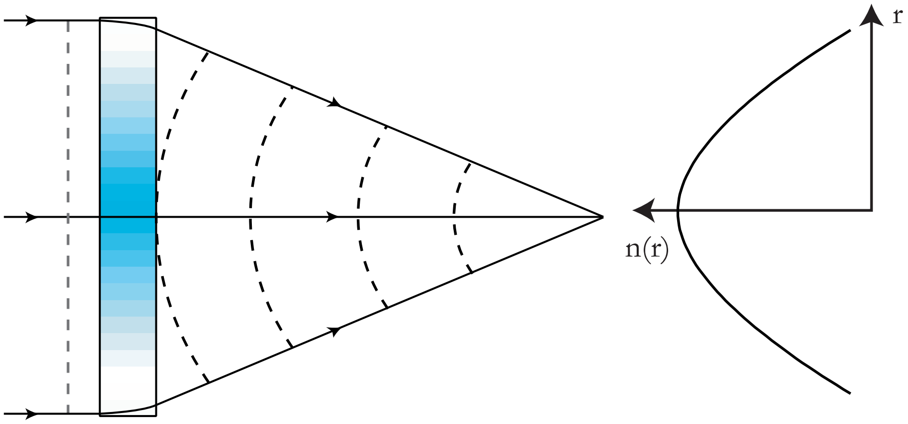

Figure 1.3. A Wood lens has planar surfaces and a radially-symmetric index gradient. The shaded region represents the index gradient, with darker regions indicating higher refractive indices.

1.5. Applications of GRIN Designs

The axial and radially-symmetric gradient-index geometries have been used for a number of designs. Gradient-index designs have been shown to reduce the number of required optical elements, or otherwise increase performance for imaging systems, including: collimators[22], camera lenses[23], binocular objectives[24,25], microscope objectives[26-28], CD lenses[29,30], eyepieces[31], endoscopes[32], and night-vision goggles[33]. GRIN designs have also been effective for diode laser couplers[34-36], fiber couplers[37-39], and lens array applications[40-42]. Other non-imaging designs using GRIN materials include Schmidt telescope corrector plates[43] and axicons[44]. Finally, there are potential uses for gradient index profiles ophthalmic lenses[15,45-47], including contact lenses[48]. Gradient-index use in ophthalmic lenses is discussed further in Progressive Addition Lens Overview.

1.6. Index Gradient Fabrication

1.6.1. Glass

There are a variety of ways to fabricate gradient-index materials. Perhaps best known are gradient-index glasses. The different fabrication methods have different index gradient sizes and depths, summarized in Table 1.1, after Moore.[49]

Method	Spatial Extent (mm)	Δn (max)
Ion Exchange	10	0.04
Neutron Irradiation	0.1	0.02
Chemical Vapor Deposition	0.1	0.01
Polymerization Techniques	100	0.01
Ion Stuffing	50	0.04
Crystal Growing	20	0.05

Table 1.1. Methods for GRIN glass fabrication, with approximate values for maximum gradient depth and Δn. (after Moore[49])

Neutron irradiation is a method in which a boron-rich glass is bombarded with neutrons to create a change in the boron atoms and a consequent change in the refractive index. Via masking, a three-dimensional index variation can be achieved.

However, heavy neutron doses are required and there are doubts about the stability and radiation hazards of the material produced.

Chemical vapor deposition (CVD) is primarily used to fabricate fiber optics with radial index gradients. A glass or quartz tube has gases passed through it, causing glass to form inside the tube, from the tube edge inward. By varying the gas compositions, an index-gradient can be created. This rod is the preform; it is then drawn out to make a fiber. Though useful for fiber manufacture, CVD is unsuitable for large-diameter elements.

For ion exchange, a base glass is immersed in a molten salt bath. Cations from the bath exchange with the alkali ions in the salt (composing the glass). This is a proven technology, with decades of study, and is used for commercial products. Also, design-for-manufacture methods are becoming available.[50] However, it requires high-temperature ovens to melt the salt bath, and so is energy intensive.

Ion stuffing uses special glass, which phase separates when heated. One phase of the glass is soluble in acid and can be dissolved out to leave a glass sponge. When this is exposed to a bath containing suitable ions or molecules, these diffuse in, creating an index gradient.

With the crystal growing method, a seed crystal is used to pull a larger crystal from a bath containing a mixture of chemicals. As the crystal is pulled, the amount of one chemical in the bath decreases. This causes the crystal to take more of a second chemical, creating a change in refractive index. This is potentially useful for working with infrared optics.

These methods are all useful for various applications. But as is seen, most glass-based GRIN materials possess a relatively small Δn, typically less than 0.04, and a small gradient extent, typically less than 10 mm, making them ill-suited for use in ophthalmic lenses.

1.6.2. Plastic

As with the gradient-index glasses, there are several ways to make gradient-index plastics. The different fabrication methods have different index gradient sizes and depths, summarized in Table 1.2. Because polymers have refractive indices ranging from 1.4 to 1.6[51,52], it is expected that a large range of maximum Δn can be achieved, and this is indeed the case.

Diffusion Method	Spatial Extent (mm)	Δn (max)
Photocopolymerization	3	0.05
Interfacial-Gel	10	0.02
Vapor Phase Transfer	5	0.03
Curved Mold	35	0.02
Immersion	40	0.1

Table 1.2. Methods for GRIN plastic fabrication, with approximate values for maximum gradient depth and Δn.

Photocopolymerization, polymerization activated by ultraviolet light, is typically done with two or more monomers mixed together in a glass tube. The tube is rotated, approximately 40 RPM, and illuminated by UV light. Only a portion of the tube is illuminated by the light, and the light is moved upward at a constant rate, approximately 1 mm/sec. The variation in the rate at which the monomers polymerize as the light travels upward leads to variation in refractive index.[53-55]

Interfacial-gel copolymerization typically uses a hollow tube made of one of the two a polymers being used for the diffusion. The tube is filled with a mixture of both monomers. The tube is heated, and the polymerization proceeds from the inside tube wall. The differing rates of polymerization of the two materials lead to an index gradient.[56]

The vapor phase transfer method places a partially polymerized rod in an inert atmosphere. The vapor of the diffusant monomer is introduced, which diffuses into the rod. After diffusion is complete, the rod is heated to complete the copolymerization.[57]

The curved mold method is a geometrical approach to creating large-radius gradient-index polymers. A partially polymerized gel is made in a mold with a curved top. Subsequently, the gel has a curved top. A second liquid monomer is added, covering the gel. The monomer diffuses into the gel; the whole of it is then heated to complete polymerization. An effective gradient-index variation is achieved through the varying composition of the sample.[58] This can also be thought of as a variation of the immersion method, described next.

For immersion copolymerization, the first monomer is partially polymerized, and the gel immersed in a liquid second monomer. After diffusion, or during diffusion, it is fully polymerized. Both radially-symmetric[51,58-60] and axial[61] index-gradients can be created. This is the method examined in this research.

Gradient-index polymers can be used to make a range of index profiles and diffusion depths. For designs requiring large Δn values, gradient-index polymers are a possible solution.

1.7. Goals and Outline of Thesis

The goal of this research is to systematically characterize the design of ophthalmic lenses with gradient-index materials and explore a polymer system for use in fabrication of large Δn gradient-index designs. This is accomplished through theoretical modeling and copolymer diffusion experiments.

Chapter 2 examines gradient-index designs for single-power ophthalmic lenses. Aberration correction is explained. Designs for minimizing distortion and reducing lens thickness are detailed.

Chapter 3 discusses concentric varifocal lenses. These are radially symmetric lenses with varying power along the lens radius. The theory for calculating the necessary index profiles for arbitrary power progressions is developed. Aberration calculations for such systems are examined.

Chapter 4 demonstrates the design and fabrication of a gradient-index axicon, using the theory from Chapter 3. This is the first gradient-index axicon fabricated. It is characterized and its performance evaluated. This also demonstrates the broader uses of this research into ophthalmic lenses.

Chapter 5 extends the gradient-index designs for progressive-power to non-rotationally symmetric designs; namely progressive addition lenses. A design approach for predicting the necessary index profile for progressive lenses is developed. Two examples are presented and their performances evaluated.

Chapter 6 addresses the need for large-extent, high-Δn index profiles, as required by the lens designs presented. To further the state of gradient-index technology, the fabrication of a gradient-index plastic is studied for a specific copolymer system. Immersion diffusion is used, and the development of the index gradient is experimentally modeled, providing the first characterization of the process in this system.

Chapter 7 includes a summary of results and suggestions for future research.

1.8. References

[1] W. Scoresby, "A description of some remarkable effects of unequal refraction," Transactions of the Royal Society of Edinburgh **11**, 8-14 (1831).

[2] S. Exner, "The Retinal Image of Insect Eyes," Sb. Akad. Wiis Wien **98**, 13 (1889).

[3] Arthur G. Bennett and Ronald B. Rabbetts, *Clinical Visual Optics*, 1 ed. (Butterworths, London, 1984).

[4] Aran Safir, "Refraction and Clinical Optics," (Harper & Row, New York, 1980), pp. 565.

[5] D. A. Atchison, "Accommodation and presbyopia," Ophthalmic & Physiological Optics **15** (4), 255-72 (1995).

[6] W. Lenne, "Visual reliability: the conception of Varilux lenses," SPIE Proceedings **601**, 24-30 (1986).

[7] G. M. Fuerter, "Ophthalmic lens design with splines," SPIE Proceedings **601**, 9-16 (1986).

[8] D. A. Atchison, "Spectacle lens design: a review," Applied Optics **31** (19), 3579-85 (1992).

[9] Tony Jarratt, "Progressive Lenses," Optical World (March), 13-16 (2000).

[10] C. M. Sullivan and C. W. Fowler, "Progressive addition and variable focus lenses: a review," Ophthalmic & Physiological Optics **8** (4), 402-14 (1988).

[11] O. Aves, UK 15,735 (1907).

[12] A. G. Poullain and D. H. J. Cornet, France (1920).

[13] Howard D. Beach, US 2,405,989 (Aug. 20 1946).

[14] Bernard Maitenaz, "Variable Power Lenses," presented at the International Ophthalmic Optical Congress, London, 1961 (unpublished).

[15] Lucian W. Bugbee, US 1,483,509 (Feb. 12 1924).

[16] A. A. Naujokas, US 3,485,556 (Dec. 23 1969).

[17] R. S. Moore, US 3,718,383 (Feb. 27 1973).

[18] J.C. Maxwell, "Solution of problems," Cambridge and Dublin Mathematics Journal **8**, 188 (1854).

[19] Rudolf Karl Luneburg, Max Herzberger, and Emil Wolf, *Mathematical theory of optics* (University of California Press, Berkeley, 1964).

[20] Duncan T. Moore, "Design of singlets with continuously varying indices of refraction," Journal of the Optical Society of America **61** (7), 886-894 (1971).

[21] R. W. Wood, in *Physical Optics* (Macmillan, London, 1905), pp. 81-90.

[22] Duncan T. Moore, “Design of single-element gradient-index collimator,” Journal of the Optical Society of America **67** (9), 1137-1143 (1977).

[23] J. B. Caldwell and D. T. Moore, “Design of gradient-index lens systems for disc format cameras,” Applied Optics **25** (19), 3351-5 (1986).

[24] J. B. Caldwell, L. R. Gardner, S. N. Houde-Walter *et al.*, “Gradient-index binocular objective design,” Applied Optics **25** (19), 3345-50 (1986).

[25] D. S. Kindred and D. T. Moore, “Design, fabrication, and testing of a gradient-index binocular objective,” Applied Optics **27** (3), 492-5 (1988).

[26] Y. Kawano, US 5,239,413 (24 August 1993).

[27] K. S. R. Krishna and A. Sharma, “Low-power gradient-index microscope objective: Design,” Applied Optics **35** (28), 5636-5641 (1996).

[28] M. Vacha, K. Hashizume, and T. Tani, “Laser scanning gradient index optics microscope: submicrometre scale spectroscopy and imaging at cryogenic temperatures,” Journal of Microscopy-Oxford **194**, 483-485 (1999).

[29] X. Yong, L. Tingdi, and Q. Jinhui, “Design of GRIN objective lens for CD system,” SPIE Proceedings **1230**, 692-693 (1990).

[30] H. Nishi, H. Ichikawa, M. Toyama *et al.*, “Gradient-index objective lens for the compact disk system,” Applied Optics **25** (19), 3340-5 (1986).

[31] J. P. Bowen, J. B. Caldwell, L. R. Gardner *et al.*, “Radial gradient-index eyepiece design,” Applied Optics **27** (15), 3170-6 (1988).

[32] T. H. Tomkinson, J. L. Bentley, M. K. Crawford *et al.*, "Rigid endoscopic relay systems: a comparative study," Applied Optics **35** (34), 6674-83 (1996).

[33] J. L. Rouke, M. K. Crawford, D. J. Fischer *et al.*, "Design of three-element night-vision goggle objectives," Applied Optics **37** (4), 622-6 (1998).

[34] R. Goering, T. Possner, and B. Messerschmidt, "Gradient index micro lenses for laser diode applications," SPIE Proceedings **2772**, 256-62 (1996).

[35] I. Kitano, H. Ueno, and M. Toyama, "Gradient-Index Lens For Low-Loss Coupling of a Laser Diode to Single-Mode Fiber," Applied Optics **25** (19), 3336-3339 (1986).

[36] J. M. Stagaman and D. T. Moore, "Laser Diode to Fiber Coupling Using Anamorphic Gradient-Index Lenses," Applied Optics **23** (11), 1730-1734 (1984).

[37] S. F. Yuan and N. A. Riza, "General formula for coupling-loss characterization of single- mode fiber collimators by use of gradient-index rod lenses," Applied Optics **38** (15), 3214-3222 (1999).

[38] R. W. Gilsdorf and J. C. Palais, "Single-Mode Fiber Coupling Efficiency With Graded-Index Rod Lenses," Applied Optics **33** (16), 3440-3445 (1994).

[39] T. Fukushima, T. Yokota, and T. Sakamoto, "Fabrication of 7 x 6 multimode optical fiber grating demultiplexer-star coupler using a single GRIN-rod lens," Journal of Lightwave Technology **15** (10), 1938-1946 (1997).

[40] S. Yamaguchi, K. Chiba, Y. Saito *et al.*, "Laser microsoldering of tape-automated bonding leads using an Nd:YAG laser end-pumped by a laser diode bar with gradient index lens array coupling," Optical Engineering **35** (12), 3585-3590 (1996).

[41] R. Blankenbecler, US 5,703,722 (1997).

[42] J. Arai, F. Okano, H. Hoshino *et al.*, "Gradient-index lens-array method based on real-time integral photography for three-dimensional images," Applied Optics **37** (11), 2034-2045 (1998).

[43] D. T. Moore, "Catadioptric system with a gradient-index corrector plate," Journal of the Optical Society of America **67** (9), 1143-6 (1977).

[44] David J. Fischer, Curtis J. Harkrider, and Duncan T. Moore, "Design and manufacture of a gradient-index axicon," Applied Optics **39** (16), 2687-2694 (2000).

[45] Gunther Guilino, Helmut Altheimer, and Herbert Pfeiffer, US 5,148,205 (Sept. 15 1992).

[46] Paul K. Manhart, Tilman W. Stuhlinger, Kenneth R. Castle *et al.*, US 5,617,252 (April 1 1997).

[47] Ronald D. Blum and Amitava Gupta, US 5,861,934 (Jan. 19 1999).

[48] Y. Koike, A. Asakawa, S. P. Wu *et al.*, "Gradient-Index Contact-Lens," Applied Optics **34** (22), 4669-4673 (1995).

[49] Duncan T. Moore, "Gradient-index optics: a review," Applied Optics **19** (7), 1035-1038 (1980).

[50] C. J. Harkrider and D. T. Moore, "Time varying boundary condition diffusion for gradient-index design," SPIE Proceedings **3482**, 780-8 (1998).

[51] Y. Ohtsuka, T. Senga, and H. Yasuda, "Light-focusing plastic and rod with low chromatic aberration," Applied Physics Letters **25** (11), 659-61 (1974).

[52] Ivan D. Nikolov and Christo D. Ivanov, “Optical plastic refractive measurements in the visible and the near-infrared regions,” Applied Optics **13** (39), 2067-70 (2000).

[53] Y. Ohtsuka, Y. Koike, and H. Yamazaki, “Studies on the light-focusing plastic rod. VI. The photocopolymer rod of methyl methacrylate with vinyl benzoate,” Applied Optics **20** (2), 280-5 (1981).

[54] Y. Koike and Y. Ohtsuka, “Studies on the light-focusing plastic rod. XV. GRIN rod prepared by photocopolymerization of a ternary monomer system,” Applied Optics **22** (3), 418-23 (1983).

[55] Y. Koike, H. Hatanaka, and Y. Ohtsuka, “Studies on the light-focusing plastic rod. XVII. Plastic GRIN rod lens prepared by photocopolymerization of a ternary monomer system,” Applied Optics **23** (11), 1779-83 (1984).

[56] Y. Koike, E. Nihei, N. Tanio *et al.*, “Graded-index plastic optical fiber composed of methyl methacrylate and vinyl phenylacetate copolymers,” Applied Optics **29** (18), 2686-2691 (1990).

[57] Y. Ohtsuka and T. Sugano, “Studies on the light-focusing plastic rod. XIV. GRIN rod of CR-39-trifluoroethyl methacrylate copolymer by a vapor-phase transfer process,” Applied Optics **22** (3), 413-17 (1983).

[58] Shang-Pin Wu, Eisuke Nihei, and Yashiro Koike, “The large dimensional radius GRIN polymer,” Polymer Journal **27** (1), 21-25 (1995).

[59] Shang Pin Wu, E. Nihei, and Y. Koike, “Large radial graded-index polymer,” Applied Optics **35** (1), 28-32 (1996).

[60] Y. Ohtsuka, "Light-focusing plastic rod prepared from diallyl isophthalate-methyl methacrylate copolymerization," Applied Physics Letters **23** (5), 247-8 (1973).

[61] Yashiro Koike, Hiroshi Hidaka, and Yasuji Ohtsuka, "Plastic axial gradient-index lens," Applied Optics **24** (24), 4321-4325 (1985).